LITERATURE CITED

1. V. S. Barashenkov, New Applications of Heavy Ions [in Russian], Atomizdat, Moscow (1977).
2. S. F. Borisov, I. G. Neudachin, et al., "Flow of rarefied gases through openings with small pressure drops," Zh. Tekh. Fiz., 43, No. 8 (1973).
3. V. D. Akin'shin and S. F. Borisov, et al., Experimental investigation of flows of rarefied gases in a capillary sieve at different temperatures," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1974).
4. B. T. Porodnov and A. G. Flyagin, "Experimental investigation of discharge of helium, neon, and argon into a vacuum through a long isolated capillary at temperatures of 295-490°K," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1978).
5. P. E. Suetin and B. T. Porodnov, et al., "Poiseuille flow at arbitrary Knudsen numbers and tangential momentum accommodation," J. Fluid. Mech., 60, Pt. 3 (1973).
6. L. M. Lund and A. S. Berman, "Flow and self-diffusion of gases in capillaries," J. Appl. Phys., 37, Parts 1 and 2 (1966).

EFFECT OF NONISOTHERMAL CONDITIONS OF SKIN FRICTION IN SUBSONIC TURBULENT CHANNEL FLOWS

D. I. Lamden, I. L. Mostinskii, and M. B. Reznikov

UDC 536.24:532.517.4

A number of different approaches are available in current literature to the theoretical determination of skin friction in nonisothermal flows in channels and pipes. Some of these (e.g., [1-3]) predict an appreciably stronger dependence of skin friction on temperature ratio $\psi = T_W/T_\infty$ (T_W is the wall temperature; T_∞ is the core fluid temperature), than those indicated by experimental results [4-7]. Significantly better agreement with existing experimental results is achieved in [8, 9] based on comprehensive numerical analysis of a system of integrodifferential equations. However, the assumptions they make are not always sound nor physically clear. Besides, the use of numerical methods does not allow the authors to relate their analysis to known limiting laws and simultaneously develop reliable numerical expressions to generalize experimental data. Physically quite clear results have been obtained in [10-12] and, in particular, very simple limiting laws for skin friction have been established. At the same time, it appears that based on the same physically clear assumptions, it is possible to obtain even more general results which agree well with experimental data. Simultaneously, these results which coincide with the limiting values at infinite Reynolds number make it possible to indicate the limits of applicability of these laws and extend them to finite Reynolds number range.

The problem is reduced to the consideration of stable turbulent fluid flow in a channel. In order to simplify the problem it is assumed that laminar and turbulent Prandtl numbers are equal to one and that the specific heat is independent of temperature. The temperature T and velocity v distributions are then described by

$$\tau = \rho(\nu^{M} + \nu^{T})|dv/dy|,\ q = -\rho c_p(\nu^{M} + \nu^{T})\ (dT/dy), \tag{1}$$

where ν^M and ν^T are molecular and eddy viscosity coefficients; ρ is the density; ν is the shear stress; q is the heat flux. Assume that τ and q vary linearly along the channel section (it will later be shown that this simplifying assumption is quite adequate for the determination of skin friction coefficients):

$$\tau = \tau_w|y|/(h/2),\ q = q_w|y|/(h/2), \tag{2}$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 69-76, January-February, 1984. Original article submitted December 14, 1982.

where τ_w and q_w are the shear stress and heat flux at the wall; y is the transverse coordinate measured from the axis of the channel with half width h/2. The dependence of ν^M and ρ on temperature is assumed to be:

$$\nu^M = \nu_w^M (T/T_w)^\alpha, \quad \rho = \rho_w (T_w/T), \tag{3}$$

where ν_w^M and ρ_w are the respective values at $T = T_w$ and α is a constant (usually $\alpha \simeq 1.5$–2.0). As far as the dependence of eddy viscosity ν^T is concerned the most rational (in any case, in the near-wall region) choice seems to be the hypothesis relating some local turbulence scale or mixing length to the distance from the wall [13]. The mathematical expression of the hypothesis on the local structure is based on the assumption of a universal relation between ν^T/ν^M and the nondimensional parameter $\eta = l\sqrt{\tau/\rho}/(\varkappa \nu^M)$, i.e., the universal function F:

$$\nu^T/\nu^M = F(\eta).$$

Here l, ρ, ν^M, and τ are local values of turbulence scale, density, molecular viscosity, and shear stress, $\varkappa \simeq 0.4$ is the Karman—Prandtl mixing length. On the other hand, the application of this hypothesis at the fluid core leads to certain doubts since the high turbulence level in this region is caused not so much by production as by diffusion of turbulent fluctuations from thin near-wall regions. In our view the synthesis of turbulent flow characteristics in the near-wall region and in the inviscid core is most successfully achieved by Reichardt equation [14] which is modified to ensure localization in the near-wall regions†:

$$\nu^T + \nu^M = \begin{cases} \nu_w^M \tilde{T}^\alpha, & 1 - |\tilde{y}| \leqslant \tilde{\delta}_l, \\ \dfrac{\varkappa}{3}\dfrac{h}{2} v_w^* \tilde{T}^{1/2}(0.5 + \tilde{y}^2)(1 - \tilde{y}^2), & 1 - |\tilde{y}| > \tilde{\delta}_l, \end{cases} \tag{4}$$

where $\tilde{T} = T/T_w$; $\tilde{y} = y/(h/2)$; $\tilde{\delta}_l = \delta_l/(h/2)$ is the nondimensional thickness of the laminar sublayer $v_w^* = \sqrt{\tau_w/\rho_w}$ is the friction velocity based on density in the near-wall region. It is possible to verify that Eq. (4) reduces to the well-known Prandtl equation for eddy viscosity in the near-wall region:

$$\nu^T/\nu^M = (l\sqrt{\tau_w/\rho})/\nu^M, \quad l = \varkappa(h/2 - |y|).$$

On the other hand, when $1 - |\tilde{y}| \ll 1$, $\tau \approx \tau_w$, and, consequently, Eq. (4) satisfies the local equilibrium condition in the near-wall region. In order to determine the laminar sublayer thickness δ_l in Eq. (4), we note that for local equilibrium requirements, the nondimensional laminar sublayer thickness η_l should be identical (usually $\eta_l = 11.5$) for isothermal as well as for nonisothermal conditions. This requirement leads to the following expression coupling $\tilde{\delta}_l$, nondimensional temperature at the laminar sublayer edge $\tilde{T}_l$. and v_w^*:

$$\tilde{\delta}_l (h/2)\, \tilde{T}_l^{1/2-\alpha} \frac{v_w^*}{\nu_w^M} = \eta_l. \tag{5}$$

Let us now consider the computation of temperature and velocity profiles using Eqs. (1) and Eqs. (2), (3), and (4). Note, firstly, that in view of the assumptions (2), velocity and temperature profiles are similar and, hence, the following expressions hold good:

$$v/v_\infty = (T - T_w)/(T_\infty - T_w), \quad q_w v_\infty/(\tau_w c_p (T_\infty - T_w)) = 1. \tag{6}$$

Integrating the second equation of the system (1) using Eqs. (6) and (4), the following expressions for temperature profiles are obtained:

$$\tilde{T}^\alpha = 1 + \alpha \frac{\mathrm{Re}_w}{2} \tilde{v}_w^{*2} (\tilde{T}_\infty - 1)(1 - |\tilde{y}|), \quad 1 - |\tilde{y}| \leqslant \tilde{\delta}_l; \tag{7a}$$

$$\tilde{T}^{1/2} = \tilde{T}_\infty^{1/2} - \frac{\tilde{v}_w^*}{2\varkappa}(\tilde{T}_\infty - 1) \ln \frac{1 + 2\tilde{y}^2}{1 - \tilde{y}^2}, \quad 1 - |\tilde{y}| > \tilde{\delta}_l. \tag{7b}$$

Here $\tilde{v}_w^* = v_w^*/v_\infty$; $\tilde{T}_\infty = T_\infty/T_w$; $\mathrm{Re}_w = v_\infty h/\nu_w^M$ is Reynolds number based on wall conditions. Equation (7a) makes it possible to express $\tilde{v}_w^*$ in terms of δ_l and $\tilde{T}_l$. In its turn $\tilde{\delta}_l$ is determined from (5) as a function of $\tilde{T}_l$. Finally,

†The double deck variant of Reichardt equation is chosen here to obtain closed form analytical expressions.

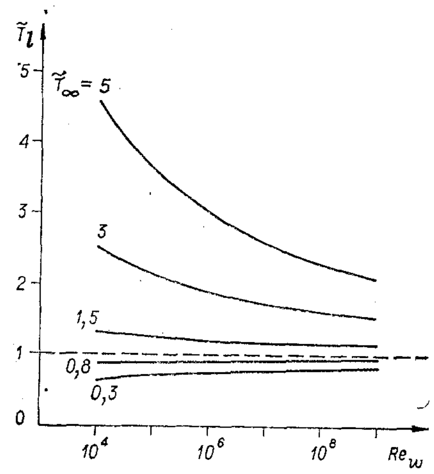

Fig. 1

$$\tilde{\delta}_l = 2\alpha\eta_l^2 \frac{\tilde{T}_\infty - 1}{\mathrm{Re}_w} \frac{\tilde{T}_l^{2\alpha-1}}{\tilde{T}_l^{\alpha} - 1}; \tag{8}$$

$$\tilde{v}_w^* = \frac{1}{\alpha\eta_l(\tilde{T}_\infty - 1)} \frac{\tilde{T}_l^{\alpha} - 1}{\tilde{T}_l^{\alpha-1/2}}. \tag{9}$$

On the other hand, from matching temperature profiles (7a) and (7b) at the edge of laminar sublayer, the following equation is obtained

$$\frac{\tilde{T}_l^{1/2-\alpha}(\tilde{T}_l^{\alpha} - 1)}{2\alpha\varkappa\eta_l(\tilde{T}_\infty^{1/2} - \tilde{T}_l^{1/2})} \ln \frac{3\mathrm{Re}_w(\tilde{T}_l^{\alpha} - 1)}{4\alpha\eta_l^2(\tilde{T}_\infty - 1)\tilde{T}_l^{2\alpha-1}} = 1 \tag{10}$$

to determine $\tilde{T}_l$ as a function of the input parameters $\tilde{T}_\infty$ and Re_w to complete the solution to the problem. Actually, having determined $\tilde{T}_l$ from (10), it is possible to find $\tilde{\delta}_l$ and $\tilde{v}_w^*$ from (8) and (9) and then determine the temperature and velocity distribution using Eqs. (6), (7a), and (7b). The results thus obtained can be reformulated in terms of Re_∞ using the obvious relation

$$\mathrm{Re}_\infty = \mathrm{Re}_w \tilde{T}_\infty^{-\alpha}.$$

The dependence of $\tilde{T}_l$ on Re_w for various values of $\tilde{T}_\infty$ ($\varkappa$ = 0.4, η_l = 11.5, and α = 1.8) is shown plotted in Fig. 1 as an example of the solution to Eq. (10).

Consider now the determination of skin friction which can be defined in two ways, with density ρ based on wall conditions or the inviscid core:

$$\tau_w = \frac{c_{fw}}{2}\rho_w v_\infty^2 = \frac{c_{f\infty}}{2}\rho_\infty v_\infty^2. \tag{11}$$

Thus, the following relation exists between skin friction coefficients based on density at the wall and in the inviscid core:

$$c_{f\infty} = c_{fw}\tilde{T}_\infty. \tag{12}$$

It follows from (11) that $c_{fw} = 2\tilde{v}_w^{*2}$. Using (9) we get

$$c_{fw} = c_{f\infty}\tilde{T}_\infty^{-1} = \frac{2}{\alpha^2\eta_l^2(\tilde{T}_\infty - 1)^2} \frac{(\tilde{T}_l^{\alpha} - 1)^2}{\tilde{T}_l^{2\alpha-1}}. \tag{13}$$

Equation (13) does not have sufficient physical clarity. In particular, there is no explicit dependence on Reynolds number, and the transition to isothermal case is not clear, when T_∞, $T_l \to 1$. In order to obtain an expression for skin friction in a more conventional and analytically convenient form, Eq. (10) is transformed using Eq. (13), resulting in the following

$$\frac{c_{fw}}{2} = H_{c_{fw}}\left[\eta_l H_{\eta l} + \frac{1}{\varkappa}\ln\left(\frac{3\mathrm{Re}_w H_{\mathrm{Re}_w}}{4\eta_l H_{\eta l}}\sqrt{\frac{c_{fw}}{2}}\right)\right]^{-2},$$

$$H_{c_{fw}}=\left(\frac{2}{\tilde{T}_{\infty}^{1/2}+1}\right)^2,\quad H_{\eta_l}=2\alpha\frac{\tilde{T}_l^{1/2}-1}{\tilde{T}_l^{\alpha}-1}\tilde{T}_l^{\alpha-1/2},\quad H_{Re_w}=2\alpha\frac{\tilde{T}_l^{1/2}-1}{\tilde{T}_l^{\alpha}-1}. \tag{14}$$

Equation (14) is identical in form to Prandtl's universal skin friction formula. The correction factors $H_{c_{fv}}$, H_{η_l}, and H_{Re_w} present in (14) approach unity as $\tilde{T}_\infty \to 1$, i.e., in the limiting isothermal flow. Thus, for the isothermal case Eq. (14) becomes

$$\frac{c_f^i}{2}=\left[\eta_l+\frac{1}{\varkappa}\ln\left(\frac{3\mathrm{Re}_w}{4\eta_l}\sqrt{\frac{c_f^i}{2}}\right)\right]^{-2}. \tag{15}$$

The same expression for c_f^i can be obtained directly by using Reichardt equation for isothermal flow.

Equation (15) differs from Prandtl's skin friction formula only by the factor 3/4 within the logarithmic sign which makes it possible to take into account the turbulence level in the inviscid core more accurately than it is done on the basis of logarithmic velocity profile. Skin friction coefficients computed from Eq. (15) do not differ by more than 3% from those obtained from the well-known empirical formula [15].

As an example of using these relations, consider the extensively studied problem of limiting skin friction coefficients (as $Re \to \infty$) and their agreement with existing experimental data. The simplest of these

$$c_{f\infty}/c_{f\infty}^i=\left(\frac{2}{\tilde{T}_\infty^{-1/2}+1}\right)^2 \quad \text{for}\quad \mathrm{Re}_\infty\to\infty, \tag{16}$$

obtained in [10] is based on turbulent boundary-layer theory with negligible viscosity and is also applicable to the present case. Here $c_{f\infty}^i$ is the skin friction coefficient (15) for isothermal flow with Reynolds number Re_∞ based on core conditions. Actually, as $Re_\infty \to \infty$ (consequently, also $Re_w \to \infty$) the temperature T_l at the edge of the laminar sublayer approaches unity, as seen from Eq. (10). Hence the corrections $H_{\eta l}$ and H_{Re_w} in (14) also tend to unity. Thus, in the present limiting case, Eq. (14) becomes

$$\frac{c_{fw}}{2}=\left[\eta_l+\frac{1}{\varkappa}\ln\left(\frac{3\mathrm{Re}_w}{4\eta_l}\sqrt{\frac{c_{fw}}{2}}\right)\right]^{-2}\left(\frac{2}{\tilde{T}_\infty^{1/2}+1}\right)^2\to(\ln\mathrm{Re}_w)^{-2}\left(\frac{2}{\tilde{T}_\infty^{1/2}+1}\right)^2. \tag{17}$$

It follows from Eq. (15) that

$$c_{fw}^i/2\to(\ln\mathrm{Re}_w)^{-2}\quad \text{for}\quad \mathrm{Re}_w\to\infty,$$

and hence we finally obtain from Eq. (17) and (12) the relations [12]:

$$c_{f\infty}/c_{fw}^i=\tilde{T}_\infty c_{fw}/c_{fw}^i=\left(\frac{2}{\tilde{T}_\infty^{-1/2}+1}\right)^2, \tag{18}$$

where c_{fw}^i is the skin friction coefficient for the isothermal flow with Reynolds number equal to Re_w. Finally, since $\ln Re_\infty = \ln(Re_w\tilde{T}_\infty^{-\alpha}) \to \ln Re_w$ in the present limit and by writing the following asymptotic series

$$c_{f\infty}^i/2\to(\ln\mathrm{Re}_\infty)^{-2}\to(\ln\mathrm{Re}_w)^{-2}\to c_{fw}^i/2,$$

we arrive at the limiting formula for skin friction in the form (16). The intermediate position in the asymptotic series transforming (18) to (16) is occupied by the limiting law recommended in [11, 16] from considerations of best fit with experiment. It is suggested there that to obtain c_f^i, Reynolds number should be determined on the basis of density at the core and viscosity at the wall. It follows from the above that the skin friction at finite Reynolds numbers should be most accurately described by Eq. (18) and least accurately by Eq. (16). This is clearly seen from Figs. 2 and 3 where accurate computations based on Eqs. (10) and (13) are compared with limiting values (18) and (16) (dashed dotted lines in Figs. 2 and 3 refer to limiting laws (18) and (16), respectively; $\alpha = 1.8$ in Fig. 2 and $\alpha = 1.64$ in Fig. 3). In particular, only at absolutely unrealistic Reynolds numbers ($Re_\infty \gtrsim 10^{20}$) does Eq. (16) quite accurately reflect the behavior of c_f. At the same time, Eq. (18) approaches suggested numerical values even in the range $Re_w \gtrsim 10^5$–10^6. However, even Eq. (18) can also

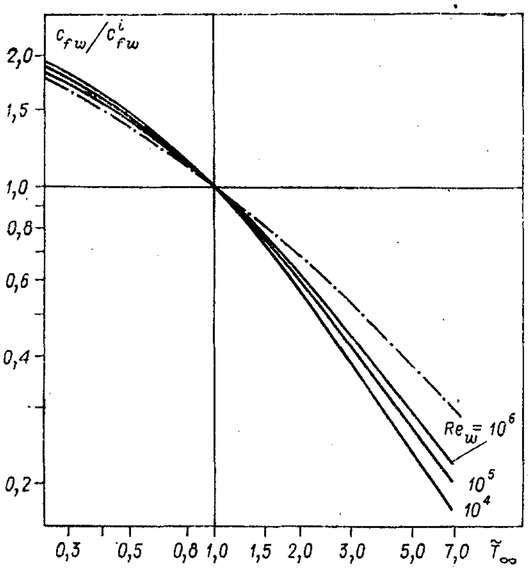

Fig. 2

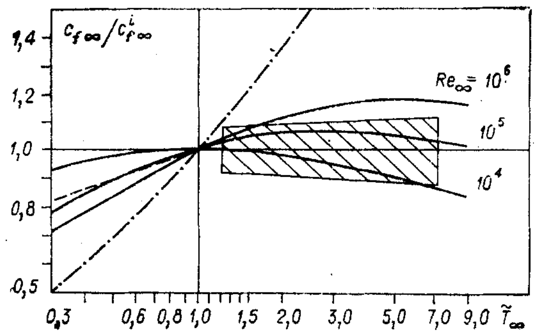

Fig. 3

lead to errors of the order of 50-100% (see Fig. 2) in the case of cooling with $Re_w \lesssim 10^4$-10^5 and $\tilde{T}_\infty \gtrsim 5$.

A comparison of Figs. 2 and 3 shows that it is more convenient to refer to wall temperature $\tilde{T}_w$ in obtaining the values of physical quantities involved in the determination of c_f. In this case, equations for c_{fw} as a function of $\tilde{T}_\infty$ are monotonic and regular, i.e., their qualitative nature does not change at different Reynolds numbers. In addition, the deviation of curves with Reynolds numbers is considerably less when referred to wall conditions (though, according to the above discussions, this deviation can reach appreciable magnitudes).

Unfortunately, in most experiments, the physical parameters are referred to fluid core conditions. It made the interpretation of these results difficult. In particular, experimental results at different Re_∞ have frequently, and without any basis, exhibited contradictions with each other and the existing theoretical concepts (see, e.g., [5]). Figure 3 can be used to explain the cause for such a "confusion" in different experimental results. Actually, in low Reynolds number ($Re_\infty \lesssim 10^5$) flows with cooling, an increase in $\tilde{T}_\infty$ initially leads to a gradual increase in $c_{f\infty}/c_{f\infty}^{i}$ and after reaching a maximum it starts decreasing, becoming less than one at very realistic values of temperature ratio $\tilde{T}_\infty \sim 3$-4. At $Re_\infty = 10^4$ the curve is always below one. Thus, for given Reynolds numbers and temperature ratio, c_f decreases with increase in $\tilde{T}_\infty$. This conclusion is confirmed by results of [6] (Re_∞ = 4000-10,000). It is necessary to mention that the accuracy of existing experimental results, as a rule, does not exceed 10-12%. Hence the authors of [4, 5] who conducted tests in the widely studied range $Re_\infty = 10^4$-10^6 came to the conclusion that $c_{f\infty}$ is independent of $\tilde{T}_\infty$. As an example the experimental results [5] are shown by the hatched region in Fig. 3. A good agreement is observed with theoretical curves for the same range of Reynolds numbers 10^4-$4 \cdot 10^5$ used in the experiments. On the other hand, an increase in skin friction coefficient has been observed in [17-19] with an increase in $\tilde{T}_\infty$, which also follows from Fig. 3 at sufficiently large values of Re_∞. Thus, in the case of cooling, depending on conditions, it is possible to observe an increase in resistance with an increase in $\tilde{T}_\infty$.

As regards heating the flow ($\tilde{T}_\infty < 1$), a decrease in $\tilde{T}_\infty$ should, as a rule, be accompanied by a decrease in $c_{f\infty}$ according to Fig. 3. A good agreement (accurate within 10%) is observed with the well-known empirical formula [7]:

$$c_{f\infty}/c_{f\infty}^{i} = \tilde{T}_\infty^{0.16},$$

obtained for the range $Re_\infty = 10^5$-10^6 (dashed line in Fig. 3). As already mentioned above, the limiting formula (18) is fulfilled quite accurately even at finite Reynolds numbers Re_∞ in the case of heating. Good agreement with many experimental results for this condition (heating) [11] can be considered a confirmation of the analysis presented here.

The transformation carried out here for skin friction in the form (13) for nonisothermal flow into an expression written in the form (14), therefore, makes it possible to establish complete agreement of this solution with the known limiting laws for isothermal as well as nonisothermal flow conditions. Besides, Eq. (14) makes it possible to express more clearly the nature of the dependence of results on one or the other parameter. At the same time the determination of corresponding corrections (excluding the already mentioned limiting cases $\tilde{T}_\infty \to 1$ or $Re \to \infty$) requires the determination of the quantity $\tilde{T}_1$ from (10). In view of this, the practical computations can be more simply carried out directly from Eq. (13) for c_f with $\tilde{T}_l$ from Eq. (10). The volume of computations is considerably reduced in this case and there is no need to solve Eq. (14) for c_{fw} (or to use corresponding approximations).

LITERATURE CITED

1. B. S. Petukhov, L. G. Genin, and S. A. Kovalev, Heat Transfer in Nuclear Power Reactors [in Russian], Atomizdat, Moscow (1974).
2. B. S. Petukhov and V. N. Popov, "Theoretical computations of heat transfer and skin friction in turbulent pipe flow of a incompressible fluid with variable physical properties," Teplofiz. Vys. Temp., 1, No. 1 (1963).
3. E. Van Driest, Turbulent Compressible Boundary Layer [Russian translation], Vol. 1(11), Mekhanika, IL, Moscow (1952).
4. A. A. Ter-Oganes'yants and S. N. Shorin, "Heat transfer and skin friction in high-temperature gas flow," Teploenergetika, No. 2 (1965).
5. N. I. Artamanov, Yu. I. Danilov, et al., "Experimental investigation of local heat transfer and skin friction in a pipe with cooling," Teplofiz. Vys. Temp., 8, No. 6 (1970).
6. N. M. Belyanin, "Experimental study of skin friction and heat transfer in pipe flow," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1964).
7. V. A. Lel'chuk and B. V. Dyadyakin, "Heat transfer from wall to turbulent air flow in a pipe and skin friction at high temperatures," in: Heat-Transfer Problems [in Russian], Izv. Akad. Nauk SSSR, Moscow (1959).
8. P. L. Maksin, B. S. Petukhov, and A. F. Polyakov, "Computation of turbulent heat and momentum transfer in incompressible, variable property fluid flow in a pipe," in: Convective and Radiative-Conductive Heat Transfer [in Russian], Nauka, Moscow (1980).
9. V. N. Popov, "Computation of heat transfer and turbulent flow processes of an incompressible fluid in circular pipe," Teplofiz. Vys. Temp., No. 4 (1977).
10. S. S. Kutateladze, Fundamentals of Heat Transfer [in Russian], Mashgiz, Moscow—Leningrad (1962).
11. A. I. Leont'ev and B. P. Mironov, "Propagation of limiting relative skin friction and heat-transfer laws in nonisothermal gas flow at finite Reynolds numbers," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1965).
12. S. S. Kutateladze and A. I. Leont'ev, Heat and Mass Transfer and Skin Friction in Turbulent Boundary Layer [in Russian], Énergiya, Moscow (1972).
13. V. M. Ievlev, High-Temperature Turbulent Flow of a Continuous Medium [in Russian], Nauka, Moscow (1975).
14. H. Reichardt, "Vollständige Darstellung der turbulenten Geschwindigkeitsverteilung in glatten Leitungen," ZAMM, 31, 208 (1951).
15. G. K. Filonenko, "Hydraulic resistance in pipes," Teploénergetika, No. 4 (1954).
16. S. S. Kutateladze and B. P. Mironov, "Relative effect of temperature factor on turbulent boundary layer at finite Reynolds numbers," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1970).
17. A. A. Gukhman, N. V. Il'yukhin, et al., "Experimental study of heat transfer and skin friction in subsonic region," Proc. Central Scientific Research I. Polzunov Institute for the Design and Planning of Boilers and Turbines, 5, No. 21 (1951).
18. B. Pinkel, "A summary of NACA research on heat transfer and friction for air flowing through tube with large temperature difference," Trans. Amer. Soc. Mech. Engrs. (ASME), 76, No. 2 (1954).
19. L. V. Humble, W. H. Lowdermilk, and L. G. Desmon, "Measurements of average heat transfer and friction coefficients for subsonic flow of air in smooth tubes at high surface and fluid temperatures," NACA Report, No. 1020 (1950).